

Supplementary material for “Clustering of multivariate tail dependence using conditional methods”

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Abstract

This supplementary materials contains additional plots for the motivating data application in Section 4 of the main text. These include elbow plots for choosing the number of clusters, estimated clusters for $k = 2$ and $k = 4$, and threshold stability plots for the conditional extremes model fits at five specified locations.

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S1 Elbow plots

Figure S1 gives the “elbow” plots of the total within-group sum of squares (TWGSS) against the number of clusters k , as described in Section 2.4 of the main text, for the motivating data application in Section 4. These are shown for the estimated clusters obtained using the dissimilarity matrix for wind speed conditional on precipitation, precipitation conditional on wind speed, and the aggregated dissimilarity matrix M as described in Equation (13) of the main text.

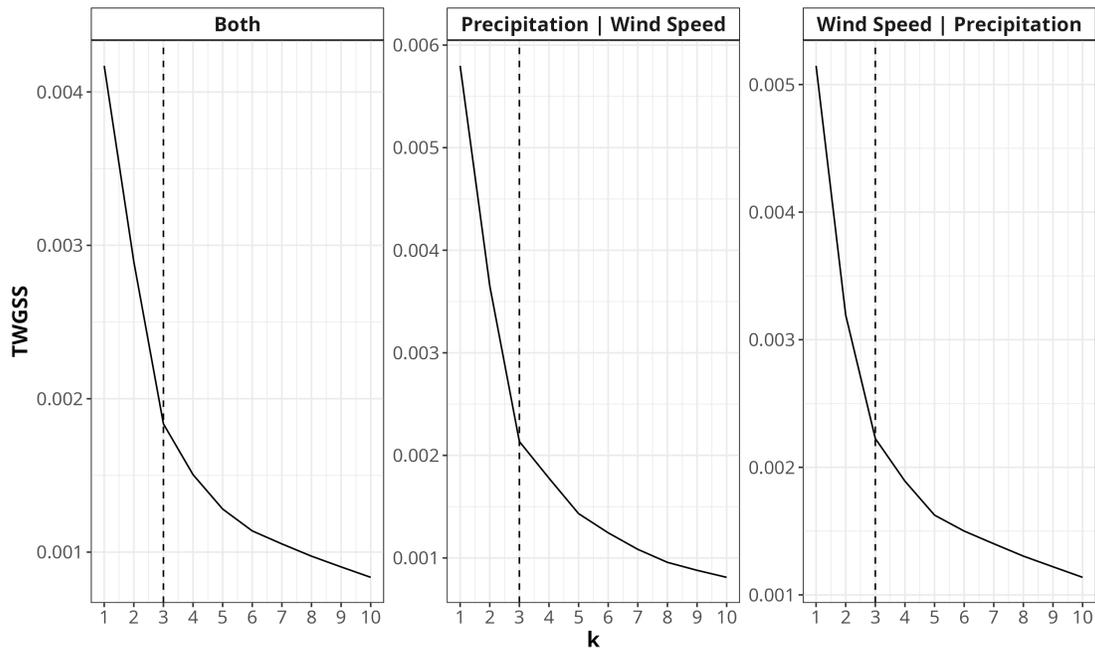


Figure S1: “Elbow” plots of the total within-group sum of squares (TWGSS) against the number of clusters k for the estimated clusters obtained using the dissimilarity matrix for wind speed conditional on precipitation (centre), precipitation conditional on wind speed (right), and the combined dissimilarity matrix (left).

S2 Estimates for $k = 2$ and $k = 4$ clusters

Figure S2 shows the estimates for $k = 2$ and $k = 4$ clusters, obtained using the aggregated dissimilarity matrix M , as mentioned in Section 4.3 of the main text.

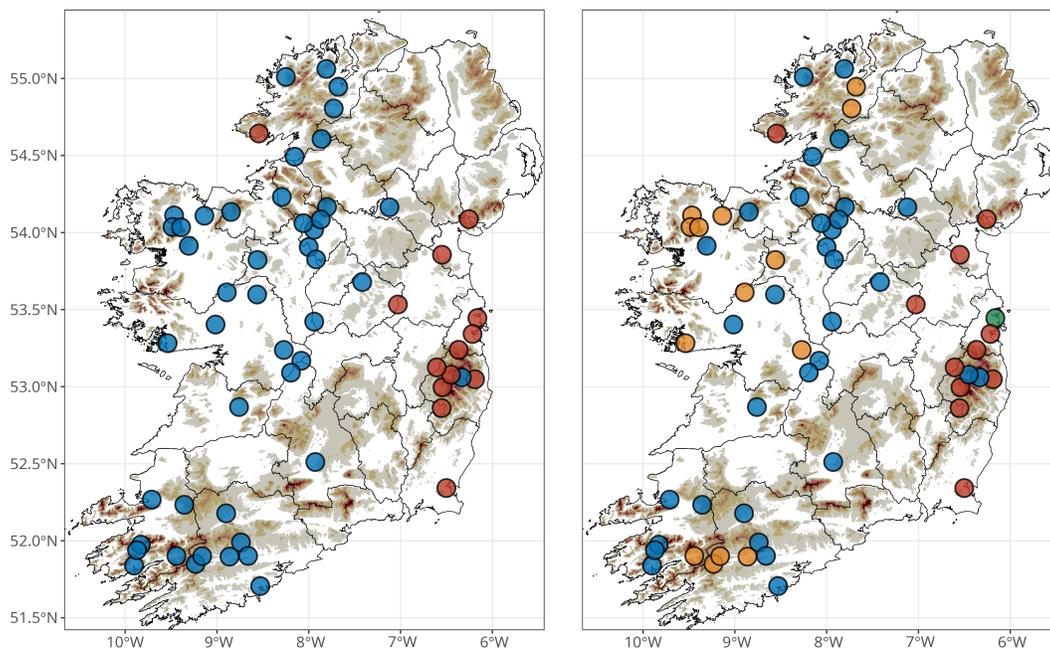


Figure S2: *Extremal cluster estimates using the combined dissimilarity matrix for $k = 2$ (left) and $k = 4$ (right) clusters. Sites are coloured by cluster label.*

S3 Threshold stability plots

Here, we present threshold stability plots for the conditional extremes model fits at the five locations highlighted in Figure 6 of the main text, or specifically, Malahide Castle and Ringsend, both in county Dublin, Glenmacnass in county Wicklow, Kilcar in county Donegal, and Derryhillagh in county Mayo.

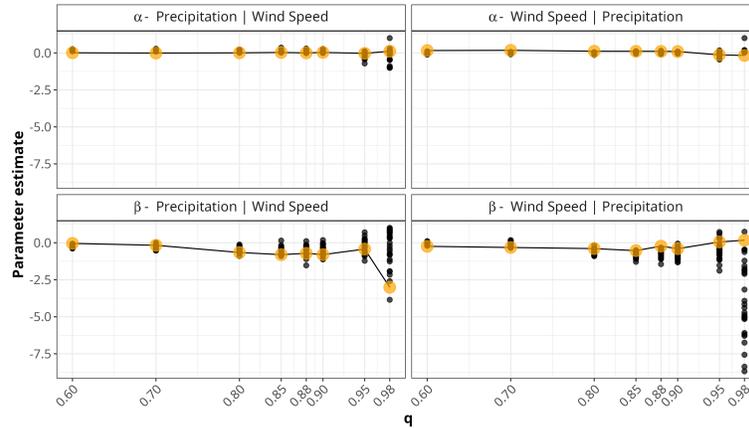


Figure S3: *Threshold stability plots for estimated model parameters $\alpha_{|i,s}$ (top) and $\beta_{|i,s}$ (bottom) for precipitation conditional on wind speed (left) and wind speed conditional on precipitation (right) at Malahide Castle, county Dublin. Estimates for 500 bootstrap samples at each quantile level q of the standard Laplace distribution are shown as black points, with yellow points indicating the estimates obtained using the full data set at each threshold.*

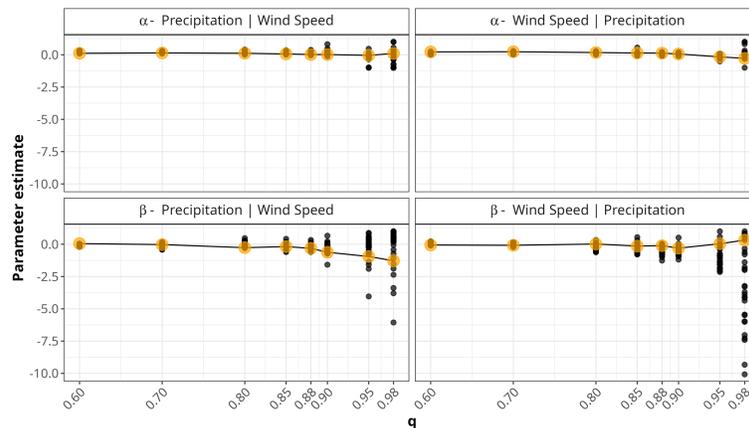


Figure S4: *Threshold stability plots for estimated model parameters $\alpha_{|i,s}$ (top) and $\beta_{|i,s}$ (bottom) for precipitation conditional on wind speed (left) and wind speed conditional on precipitation (right) at Ringsend, county Dublin. Estimates for 500 bootstrap samples at each quantile level q of the standard Laplace distribution are shown as black points, with yellow points indicating the estimates obtained using the full data set at each threshold.*

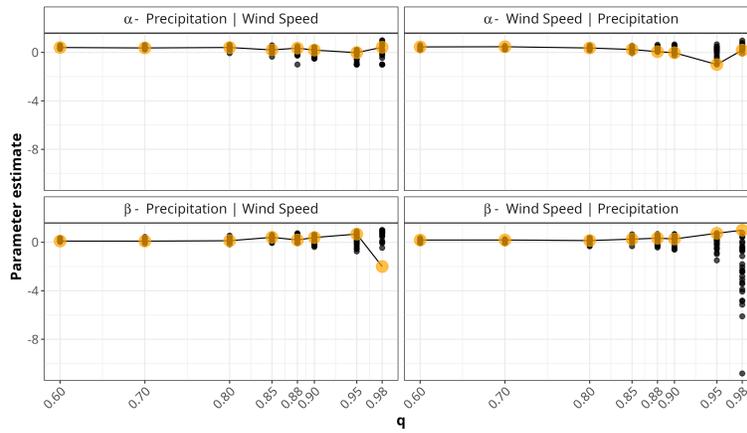


Figure S5: *Threshold stability plots for estimated model parameters $\alpha_{|i,s}$ (top) and $\beta_{|i,s}$ (bottom) for precipitation conditional on wind speed (left) and wind speed conditional on precipitation (right) at Glenmacnass, county Wicklow. Estimates for 500 bootstrap samples at each quantile level q of the standard Laplace distribution are shown as black points, with yellow points indicating the estimates obtained using the full data set at each threshold.*

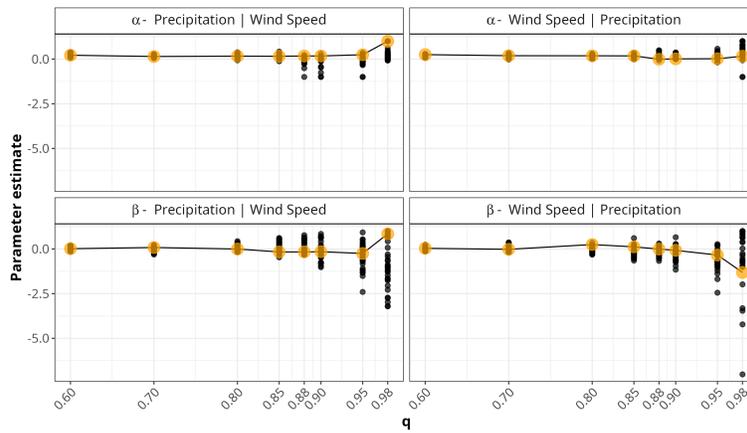


Figure S6: *Threshold stability plots for estimated model parameters $\alpha_{|i,s}$ (top) and $\beta_{|i,s}$ (bottom) for precipitation conditional on wind speed (left) and wind speed conditional on precipitation (right) at Kilcar, county Donegal. Estimates for 500 bootstrap samples at each quantile level q of the standard Laplace distribution are shown as black points, with yellow points indicating the estimates obtained using the full data set at each threshold.*

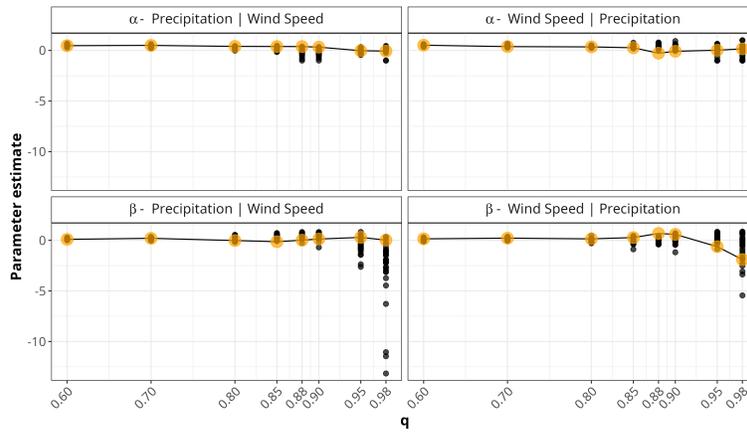


Figure S7: *Threshold stability plots for estimated model parameters $\alpha_{|i,s}$ (top) and $\beta_{|i,s}$ (bottom) for precipitation conditional on wind speed (left) and wind speed conditional on precipitation (right) at Derryhillagh, county Mayo. Estimates for 500 bootstrap samples at each quantile level q of the standard Laplace distribution are shown as black points, with yellow points indicating the estimates obtained using the full data set at each threshold.*